Reg. No. : | | | | | | | | | | | | |

**Question Paper Code : 27327**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the partial differential equation of all spheres whose centres lie on the Z – axis, by the elimination of arbitrary constants.

2. Solve $(D+D'-1)(D-2D'+3)z=0$.

3. Find the root mean square value of $f(x)=x(l-x)$ in $0\le x\le l$.

4. Find the sine series of function $f(x)=1$, $0\le x\le \pi$.

5. Solve $3x\frac{\partial u}{\partial x}-2y\frac{\partial u}{\partial y}=0$; by method of separation of variables.

6. Write all possible solutions of two dimensional heat equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

7. If $F(s)$ is the Fourier Transform of $f(x)$, prove that $F\{f(ax)\}=\frac{1}{a}F\left(\frac{s}{a}\right), a\ne 0$.

8. Evaluate $\int_0^\infty \frac{s^2}{(s^2+a^2)(s^2+b^2)}ds$ using Fourier Transforms.

9. Find the Z – transform of $\frac{1}{n+1}$.

10. State the final value theorem. In Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find complete solution of $z^2(p^2+q^2)=(x^2+y^2)$. (8)

(ii) Find the general solution of $(D^2+2DD'+D'^2)z=2\cos y - x\sin y$. (8)

Or

(b) (i) Find the general solution of $(z^2-y^2-2yz)p+(xy+zx)q=(xy-zx)$. (8)

(ii) Find the general solution of $(D^2+D'^2)z=x^2y^2$. (8)

12. (a) (i) Find the Fourier series expansion the following periodic function of period 4 $f(x)=\begin{cases}2+x & -2\le x\le 0\\ 2-x & 0<x\le 2\end{cases}$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$. (8)

(ii) Find the complex form of Fourier series of $f(x)=e^{ax}$ in the interval $(-\pi,\pi)$ where $a$ is a real constant. Hence, deduce that $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{a^2+n^2}=\frac{\pi}{a\sinh a\pi}$. (8)

Or

(b) (i) Find the half range cosine series of $f(x)=(\pi-x)^2, 0<x<\pi$. Hence find the sum of series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots$ (8)

(ii) Determine the first two harmonics of Fourier series for the following data. (8)

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|---|---|---|---|---|---|---|
| $x$ : | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ |
| $f(x)$ : | 1.98 | 1.30 | 1.05 | 1.30 | –0.88 | –0.25 |

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $v=\begin{cases}\frac{2kx}{l} & in\ \ 0<x<\frac{l}{2}\\ \frac{2k(l-x)}{l} & in\ \ \frac{1}{2}<x<l\end{cases}$. Find the displacement of the string at any distance $x$ from one end at any time $t$. (16)

Or

(b) A bar 10 cm long with insulated sides has its ends A and B maintained at temperature $50°C$ and $100°C$, respectively, until steady state conditions prevails. The temperature at A is suddenly raised to $90°C$ and at the same time lowered to $60°C$ at B. Find the temperature distributed in the bar at time $t$. (16)

14. (a) (i) Find the Fourier sine integral representation of the function $f(x)=e^{-x}\sin x$. (8)

(ii) Find the Fourier cosine transform of the function $f(x)=\frac{e^{-ax}-e^{-bx}}{x}, x>0$. (8)

Or

(b) (i) Find the Fourier transform of the function $f(x)=\begin{cases}1-|x|, & |x|\le 1\\ 0, & |x|>1\end{cases}$

Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$. (8)

(ii) Verify the convolution theorem for Fourier transform if $f(x)=g(x)=e^{-x^2}$. (8)

15. (a) (i) If $U(z)=\frac{z^3+z}{(z-1)^3}$, find the value of $u_0$, $u_1$ and $u_2$. (8)

(ii) Use convolution theorem to evaluate $z^{-1}\left\{\frac{z^2}{(z-3)(z-4)}\right\}$. (8)

Or

(b) (i) Using the inversion integral method (Residue Theorem), find the inverse Z- transform of $U(z)=\frac{z^2}{(z+2)(z^2+4)}$. (8)

(ii) Using the Z- transform solve the difference equation $u_{n+2}+4u_{n+1}+3u_n=3^n$ with $u_0=0, u_1=1$. (8)